

# The Dynamics of Time-Consistent Redistributive Capital Taxation

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Studies on optimal capital taxation and redistribution within the neoclassical growth model found that the equilibrium tax rate is not zero when the government cannot commit to its future policy. This paper investigates the dynamic properties of time-consistent Markovian tax strategies. It shows that the optimal tax rate is not constant over time unless the economy is at the steady-state, that adjustment dynamics of tax policy are unique, and that the optimal tax rate and the share of GDP redistributed by the government rise as the economy develops. Therewith the paper provides a simple theory of growth of the public sector.

*Keywords:* Dynamic Optimal Taxation, Redistribution, Markov-Perfect Equilibrium.

*JEL:* C73, H21, O40, E60.

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## 1. INTRODUCTION

The economic literature has provided several alternative and complementing explanations for a non-zero tax rate on capital income. One explanation is that suppliers of labor (workers) and capital (capitalists) are involved in a struggle of income distribution which is represented by a government that maximizes weighted welfare of workers and capitalists using redistributive taxation. In his seminal article, Judd (1985) has shown that if a government can credibly commit to a path of tax policy, it will generally set the equilibrium tax rate to zero irrespective of the income distribution and the weight of workers in the social welfare function. Away from the steady-state, however, the optimal tax rate is generally not zero. In the standard model, the government sets the tax rate to its maximum value at the beginning and commits that taxes converge towards zero in the future. For a developing economy this implies that taxes and transfers are the lower the higher the level of economic development.<sup>1</sup>

In such games of optimal taxation the government acts as a Stackelberg leader and the private sector as a follower and the open-loop solution (where the government selects a time path of taxation at the beginning of the planning period) is generally time-inconsistent (See e.g. Chamley, 1986 and Xie, 1997). Lansing (1999) and Xie (1997), however, discuss an important counter example: when utility functions are logarithmic the optimal tax rate under commitment is not zero and the announced policy is time-consistent. In this special case capitalists' consumption is a linear function of the current capital stock and the path of consumption cannot be controlled independently from capital. In other words, the open-loop solution coincides with the Markovian solution (sometimes also called feedback

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<sup>1</sup>See Jones et al. (1993) and Frankel (1998) for discussion of adjustment dynamics of taxation in Chamley's (1986) representative agent model of optimal capital taxation. Kemp et al. (1993) show that the equilibrium under commitment allows also for a variety of non-standard tax dynamics when the assumption of a constant elasticity of marginal utility is relaxed. For example, the equilibrium can be completely unstable, or the optimal trajectory can contain closed orbits both of which may imply that the equilibrium tax of zero is never reached. Lansing (1999) contains a short overview of complementing explanations of non-zero capital taxation. See Pohjola (1983) for alternative solution concepts for the dynamic game of redistribution between classes.

solution) where the government sets the tax rate depending on the state of the economic system as reflected by the stock of capital.<sup>2</sup>

Optimal capital taxation based on time-consistent Markovian strategies has been investigated by Kemp et al. (1993) in continuous time and by Krusell (2002) in discrete time. Given Markovian strategies the optimal equilibrium tax rate is generally not zero since without commitment the government will always have an incentive to deviate from the zero tax policy. The dynamics of optimal time-consistent taxation and redistribution, however, have not yet been discussed. Kemp et al. (1993) assume that the Markovian equilibrium is stable and Krusell (2002) restrict his analysis to a special case with one hundred percent depreciation of capital where the optimal tax rate is constant over time. This paper extends the literature of optimal capital taxation and redistribution by a discussion of adjustment dynamics. For the special case of logarithmic utility I provide a qualitative discussion which is supplemented by numerical computations of adjustment paths for the more general case of iso-elastic utility.

The main finding is that the adjustment path is unique and that the optimal tax rate is – in contrast to the policy outcome under commitment – an increasing function of the capital stock and therewith positively correlated with the level of income produced in the economy. Moreover, even when the optimal capital tax rate is positive in the equilibrium (as it turns out to be, for example, in the case of log-utility), it may become negative when the economy is far below its steady-state. In this case income transfers run from workers to capitalists in order to enhance investment and to induce faster growth.

The finding that optimal tax rates are increasing in income provides an explanation for the observation that the size of the public sector is positively correlated with the level of economic development (See e.g. Boix, 2001, for a survey of the literature and a recent panel analysis). It complements existing explanations like Wagner’s Law (Ram, 1987) or Baumol’s Cost Disease (Baumol, 1967). While these studies apply broader definitions of

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<sup>2</sup>An alternative, complementing approach to establish time-consistency has been followed by Benhabib and Rustichini (1997). There, the loss of credibility resulting from deviation of an announced tax path is used as a threat that triggers either the second best or a certain third best taxation policy in a representative agent Chamley (1986)-type model.

the public sector the current paper follows Meltzer and Richard (1981) and uses the share of income redistributed by government as the measure of the relative size of government. In a related paper Krusell and Ríos- Rull (1999) investigate the effect of inequality on the size of the public sector. Their politico-economic framework is much more complex than the one investigated here. Consequently, their analysis is confined to numerical computations of steady-state tax rates and transfers while the current paper focusses on the role of taxation and redistribution in economic development over time. Lindner and Strulik (2003) investigate a Markovian Stackelberg strategy of taxation and redistribution in a differential game of Alesina and Rodrik's (1994) median voter model. In this framework of constant marginal returns on capital they find that optimal tax rates are also constant over time and there exist no adjustment dynamics neither in economic growth nor in growth of the public sector.

This paper argues that the neoclassical feature of decreasing marginal returns on capital can be employed as driving force of increasing redistribution and government size: At stages of low development when capital is relatively scarce and capital productivity is high it is in the interest of workers to renounce on transfers from taxation of capitalists (if capitalists' utility has positive weight in social welfare the government may even subsidize capital income). Higher capital income fosters investment and marginal productivity is so high that workers benefit to a large extent from rising wages generated through a rising capital stock. At stages of high development, on the other hand, capital is relatively abundant and marginal productivity and therewith the potential wage increase induced by further investment are relatively low. Now it becomes desirable for workers to extract transfers from capital income. In other words, workers prefer growth over income transfers from capitalists at low stages of development when growth opportunities are large, and income distribution over growth at higher stages of development. Consequently, when workers' preferences are decisive in the social welfare function maximized by the government, the size of the public sector increases with economic development.

## 2. THE MODEL

The description of the economy largely follows Kemp et al. (1993). The population consists of a continuum of capitalists and workers, each of measure one. Output is produced by competitive firms using labor supplied by workers and capital supplied by capitalists. The production function is of Cobb-Douglas type with capital share  $\alpha$  so that wages and interest rates are given by  $w = (1 - \alpha)k^\alpha$  and  $r = \alpha k^{\alpha-1}$ .

Capitalists maximize intertemporal utility of consumption,  $c_k(k)$ , given by  $\int_0^\infty u(c_k)e^{-\rho t} dt$ , where instantaneous utility is of the iso-elastic form  $u(c_k) = c_k^{1-\sigma}/(1 - \sigma)$  with special case  $u(c_k) = \ln(c_k)$  for  $\sigma = 1$ . They are facing the budget constraint

$$(1) \quad \dot{k} = (1 - \tau)rk - \delta k - c_k = (1 - \tau)\alpha k^\alpha - \delta k - c_k ,$$

where  $\rho$  is the rate of time preference,  $\delta$  is the rate of depreciation,  $r$  the gross interest rate, and  $\tau$  the tax rate on capital income. The solution of the utility maximization problem is characterized by the Ramsey rule  $\dot{c}_k/c_k = [r(1 - \tau) - \delta - \rho]/\sigma$ , i.e. the consumption strategy  $c_k(k)$  fulfils

$$(2a) \quad c_k'(k) = \frac{\dot{c}}{\dot{k}} = \frac{[r(1 - \tau) - \delta - \rho]c_k}{\sigma[r(1 - \tau)k - \delta k - c_k]}$$

and the transversality condition

$$(2b) \quad \lim_{t \rightarrow \infty} c_k^{-\sigma} k e^{-\rho t} = 0 .$$

Workers consume their wage income plus transfers redistributed from capital income  $c_w = w + \tau rk$ , i.e.

$$(3) \quad c_w = [1 - \alpha(1 - \tau)]k^\alpha ,$$

from which they derive intertemporal utility  $\int_0^\infty v(c_w)e^{-\rho t} dt$ , where instantaneous utility  $v(c_w) = c_w^{1-\sigma}/(1 - \sigma)$  with  $v(c_w) = \ln(c_w)$  for  $\sigma = 1$ . Note that the tax rate is not restricted to be non-negative. A negative tax rate means that the government redistributes income from workers to capitalists.

By choosing the appropriate path of taxation  $\tau(k)$ , the government maximizes weighted utility of both groups given (1), i.e. it maximizes the Hamiltonian

$$H = v([(1 - \alpha(1 - \tau)]k^\alpha) + \gamma u(c_k(k)) + \lambda[(1 - \tau)\alpha k^\alpha - \delta k - c_k(k)] \quad ,$$

where  $\gamma$  reflects the weight of capitalists and  $\lambda$  is the shadow price of capital. The special case where  $\gamma \rightarrow 0$  has an alternative interpretation. It can be regarded as a government maximizing utility of the median voter and where the median voter is a worker. After a few algebraic transformations one obtains from the first order conditions

$$(4) \quad v' = \lambda \quad ,$$

$$(5) \quad \frac{\dot{\lambda}}{\lambda} = \left[ \rho + \delta - \alpha^2(1 - \tau)k^{\alpha-1} + \left(1 - \gamma \frac{u'}{v'}\right) \frac{\partial c_k}{\partial k} - \frac{\partial c_w}{\partial k} \right] .$$

Differentiating (4) with respect to time yields  $\dot{\lambda} = v''c_w' \dot{k}$  and after substitution of  $c_w'$  derived from (3) and insertion in (5) one obtains

$$(6) \quad \frac{\dot{\lambda}}{\lambda} = -\frac{\sigma}{c_w} \left[ \alpha \frac{c_w}{k} + \frac{\alpha c_w}{1 - \alpha(1 - \tau)} \frac{\partial \tau}{\partial k} \right] \dot{k} .$$

Substituting  $\partial \tau / \partial k = \dot{\tau} / \dot{k}$ , equating with (5), and solving for  $\dot{\tau}$  verifies that the optimal strategy  $\tau(k)$  solves

$$(7) \quad \dot{\tau} = \frac{1 - \alpha(1 - \tau)}{\alpha \sigma} \left[ \alpha k^{\alpha-1} - \left(1 - \gamma \left(\frac{c_w}{c_k}\right)^\sigma\right) \frac{\partial c_k}{\partial k} - (\rho + \delta) - \alpha \sigma \frac{\dot{k}}{k} \right]$$

and the transversality condition  $\lim_{t \rightarrow \infty} \lambda k e^{-\rho t} = 0$ . The partial derivative  $\partial c / \partial k$  in (7) shows that the government takes strategic interaction into account. It prevents a general analytical solution of the problem. For the special case of logarithmic utility, however, the derivative assumes a very simple expression and the problem can be solved analytically.

3. TAXATION, REDISTRIBUTION, AND ECONOMIC DEVELOPMENT:  
ANALYTICAL SOLUTION

When instantaneous utility is logarithmic ( $\sigma = 1$ ) it is straightforward to verify that the linear consumption strategy

$$(8) \quad c_k = \rho k$$

solves (2a) and (2b). Inserting  $c_k = \rho k$ ,  $\partial c/\partial k = \rho$  and  $\sigma = 1$ , and substituting (1) into (7) yields an ordinary differential equation for the optimal tax rate:

$$(9) \quad \dot{\tau} = \frac{1 - \alpha(1 - \tau)}{\alpha} \{(\alpha + \gamma) [1 - \alpha(1 - \tau)] k^{\alpha-1} - (1 - \alpha)(\delta + \rho) - \rho\} .$$

Note that the optimal policy under Markovian strategies does not directly depend on time but on the state of the economy and is therefore given by a policy function  $\tau(k)$  that satisfies (9) together with the equation of motion (1). The shape of this policy function can be determined by phase diagram analysis.

One equilibrium of (9) is where  $1 - \alpha(1 - \tau) = 0$ , which is the case for  $\tau_{min} = -(1 - \alpha)/\alpha$ . This tax rate, however, cannot be optimal since it implies that workers consume nothing and a marginal deviation from the policy provides infinite utility. The only other tax rate realizing  $\dot{\tau} = 0$  is at

$$(10) \quad \tau^* = 1 - \frac{\alpha + \gamma}{\alpha [1 + \gamma + \rho/(\rho + \delta)]} .$$

This tax rate implies an equilibrium capital stock  $k^* = (\alpha + \gamma) / [(1 + \gamma)(\rho + \delta) + \rho]^{1/(1-\alpha)}$ , which is the solution already obtained by Kemp et al. (1993) as feedback solution and by Lansing (1999) as open-loop solution.<sup>3</sup> The value added of this section of the current article lies in the supply of the corresponding transitional dynamics.

*PROPOSITION 1. Given the problem of optimal capital taxation described above, taxes and the economy converge along a unique adjustment path towards the equilibrium. With*

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<sup>3</sup>For comparison note that Kemp et al. neglect depreciation and that Lansing puts the weight on workers utility, i.e.  $\gamma$  corresponds to  $1/\gamma$ .

rising income per capita the optimal capital tax rate increases monotonously i.e. richer economies optimally have a larger public sector. Moreover, at sufficiently low income levels the optimal capital tax rate can assume negative values implying that transfers run from workers to capitalists.

*Proof.* From (9) the  $\dot{\tau} = 0$  locus is given by

$$(11) \quad \tau(k) = \frac{(1 - \alpha)(\delta + \rho) + \rho}{\alpha(\alpha + \gamma)} k^{1-\alpha} - \frac{1 - \alpha}{\alpha} ,$$

which is an increasing function in  $k$  with  $\dot{\tau} > 0$  above and  $\dot{\tau} < 0$  below the  $\dot{\tau} = 0$ -curve and  $\tau = -(1 - \alpha)/\alpha$  for  $k = 0$ . From (1) the  $\dot{k} = 0$ -locus is given by

$$(12) \quad \tau(k) = 1 - \frac{\delta + \rho}{\alpha} k^{1-\alpha} ,$$

which yields a curve starting at  $\tau = 1$  for  $k = 0$  and decreasing in  $k$  with  $\dot{k} < 0$  above and  $\dot{k} > 0$  below the curve. The resulting phase diagram is shown in Figure 1. The equilibrium is a saddlepoint. In finite time, all trajectories with exception of the stable manifold reach either  $k = 0$  implying  $c_w = c_k = 0$  or  $\tau = \tau_{min}$  implying  $c_w = 0$  and can thus not be optimal. The optimal strategy (the policy function) is given by the stable manifold  $\tau(k)$ . Since  $\tau$  is increasing in  $k$  as the economy converges towards the equilibrium from below  $k^*$ , economies with higher capital stock – and hence income – have a larger public sector. Adjustment is monotonic since the problem is two-dimensional and the Hamiltonian is concave in states and controls.

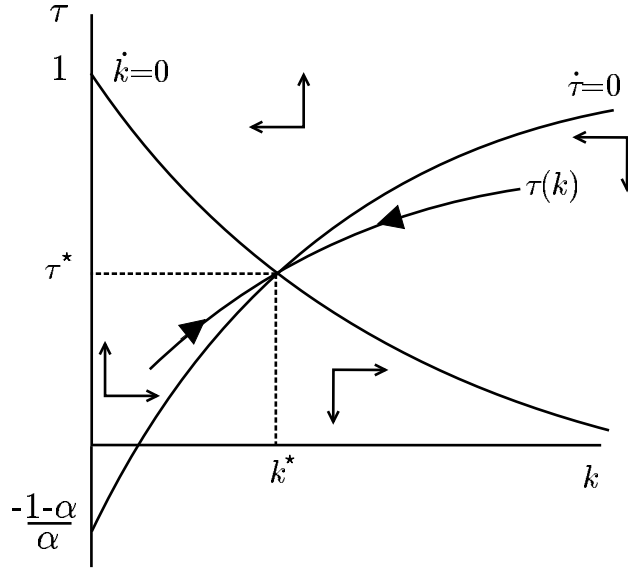
Since the  $\dot{\tau} = 0$ -locus crosses the  $k$ -axis at  $k > 0$ , the stable manifold may as well cross the  $k$ -axis at some positive  $k$  and hence the optimal tax  $\tau(k)$  may in principle become zero or negative when the capital stock is sufficiently small, i.e. the economy is at a low stage of development.

Since both tax rate and tax base ( $\alpha k^\alpha$ ) increase as the economy develops, the government sector expands in absolute terms in a developing economy. The share of GDP of the government sector given by  $\tau\alpha$  is linearly increasing in the tax rate.

□



FIGURE 1. Dynamics of Optimal Capital Taxation: Phase Diagram



As a corollary of the result that taxes increase in income we obtain:

**COROLLARY 3.1.** *As the economy develops the workers' share of income increases.*

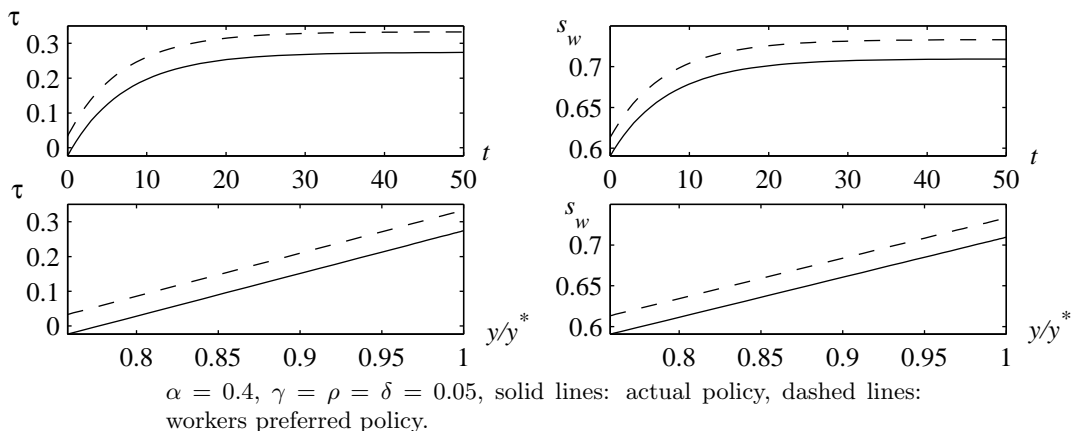
In order to see this, define workers' share of income as  $s_w \equiv c_w/k^\alpha = 1 - \alpha(1 - \tau)$  and observe that  $s_w$  is increasing in  $\tau$  which in turn is increasing in income. Since  $\partial\tau^*/\partial\gamma < 0$  (which can be obtained from (10)), a lower weight of capitalists raises the equilibrium tax rate.

To calculate adjustment dynamics for an example economy I set the capital share to 0.4 and  $\gamma$ ,  $\rho$ , and  $\delta$  to 0.05. The implied equilibrium capital tax rate is 0.27, a value which comes close to tax rates for some fully developed countries. Based on the methodology of Mendoza et al. (1994) Carey and Tchilinguirian (2000) calculate average effective capital tax rates of 0.27 for the U.S., 0.26 for Australia, and 0.24 for Japan. For OECD countries with lower income per capita they frequently obtain smaller capital tax rates of for example 0.16 for Korea and Spain, 0.13 for Greece and 0.11 for Portugal.<sup>4</sup> I compute

<sup>4</sup>The values cited originate from Table 1, capital based on gross operating surplus, period 1991-1997. Although there are also notable exceptions, the rule that higher income countries have higher capital tax

adjustment dynamics by backward integration (Brunner and Strulik, 2002) using  $k = 0.5k^*$  as termination criterion. Forward looking we consider an economy starting with half the capital stock of a fully developed economy.

FIGURE 2. Tax Dynamics and Income Distribution



Solid lines in Figure 2 show the development of taxation and income distribution. At the beginning (where the economy possesses half of its long-run capital stock) optimal tax rates are approximately zero and workers' share in income corresponds to the labor share  $s_w \approx 1 - \alpha = 0.6$ . At this stage the capital stock is sufficiently small and capital productivity is sufficiently high that worker prefer very low taxes in order to support investment and rapid growth. By setting  $\gamma$  to zero I have also calculated the policy preferred by workers. This scenario is represented by dashed lines in Figure 2 and can be interpreted as the policy outcome when the government maximizes utility of the median voter, which is assumed to be a worker. Initially, workers would prefer a mildly positive tax rate of 4 percent. Capitalists, however, always prefer the minimum tax rate  $\tau_{min} = -(1 - \alpha)/\alpha$  where workers consume nothing (This can be seen by setting  $\gamma$  to infinity in (10) ). Since capitalists enter the social welfare function with a positive weight of five

rates is supported on average. Carey and Tchilinguirian calculate a tax rate of 24.4 percent for the G7 average and of 22.0 percent for an average of 23 OECD countries. Mendoza et al. (1994) calculate effective tax rates from 1965 to 1988 for the G7 countries showing considerable increase of tax rates for all seven countries. The observation that taxes increase less during the 1980 to 1997 period investigated by Carey and Tchilinguirian supports the hypothesis that tax rates converge towards an upper bound.

percent, their preference pushes the optimal tax rate down to a slightly negative value close to zero at the beginning.

As time evolves and the economy develops, taxes and workers' income share increase. Workers' income share converges towards a value of about ten percentage points above the labor share. The lower two panels in Figure 2 show taxes and income distribution against the level of development measured by the relative distance of the economy from its steady-state per capita income  $y^* = k^{*\alpha}$ . In the left panel, one sees that taxes (and therewith income redistributed) is positively and almost linearly correlated with the economy's degree of development,  $y/y^*$ .

When we follow Meltzer and Richard (1981) and use the share of income redistributed by government as a measure of the relative size of government, the model provides a theory for the empirical observation that both relative government size and income per capita have grown in all countries of the western world during the last century (See e.g. Borchering, 1985, Boix, 2002).<sup>5</sup> The tax dynamics obtained are in sharp contrast with tax policy under commitment where the optimal tax rate is initially high and converges towards zero as the economy develops (Judd, 1985, Jones et al., 1993). Qualitatively, the results correspond to tax dynamics obtained by Hamada (1967) who has shown that optimal transfers to workers increase with the capital stock in a neoclassical model where capitalists are facing a given savings rate. Interestingly, Hamada obtains also the result that at very low levels of development optimal transfers run from workers to capitalists in order to foster capital accumulation and growth. His result is confirmed here in a considerably more complex world where capitalists' maximize intertemporal utility and governments performs time-consistent redistribution.<sup>6</sup>

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<sup>5</sup>Borchering et al. (2002) show that this pattern does no longer uniquely hold for all developed countries during the last two decades. While relative government size still grows in some countries, it stagnates or decreases in others. Stagnation on a high level could be explained within the model as completed convergence towards the steady-state. A secular fall, however, remains unexplained.

<sup>6</sup>Kaitala and Pohjola (1990) derive feedback Nash-Equilibria where the economy decays at a constant rate because workers leave capitalists with zero income. Against the background of the current paper, however, it could be conjectured that this somewhat peculiar corner solution originates from their assumption of linear utility functions.

4. TAXATION, REDISTRIBUTION, AND ECONOMIC DEVELOPMENT:  
CONSTANT ELASTICITY OF INTERTEMPORAL SUBSTITUTION

For the general case of iso-elastic utility, the first order conditions of the capitalists' maximization problem provide the Ramsey rule

$$(13) \quad \dot{c}_k = \frac{[(1 - \tau)\alpha k^{\alpha-1} - \delta - \rho]c_k}{\sigma} .$$

Hence an optimal consumption strategy  $c(k)$  fulfils

$$(14) \quad \frac{\partial c_k}{\partial k} = \frac{\dot{c}}{\dot{k}} = \frac{[(1 - \tau)\alpha k^{\alpha-1} - \delta - \rho]c_k}{\sigma [(1 - \tau)\alpha k^\alpha - \delta k - c_k]}$$

and the transversality condition (2b)

Some remarks are helpful for a correct understanding of equilibrium strategies and adjustment dynamics. To begin with, the Ramsey rule (13) does not constitute a capitalists' strategy. A pair of strategies,  $c(k), \tau(k)$ , is given by the solution of the three-dimensional system (1), (7), and (13) under consideration of (14). It cannot be analytically represented. Generally, there are infinitely many solutions (trajectories in a diagrammatic exposition as in Shimomura, 1991). From these, however, only those on the stable manifold fulfil the transversality condition by not leading to zero consumption in finite time but converging towards a non-trivial equilibrium  $(\tau^*, c_k^*, k^*)$  instead. It can be verified numerically (for parameter values used below) that the stable manifold is one-dimensional so that the solution  $c_k(k)$  and  $\tau(k)$  is unique for any given state  $k$ .

As in Kemp et al. (1993) the Ramsey rule is not derived from a HJB equation. Capitalists do not take into account the influence that their consumption behavior may have on aggregate capital and therewith on the rate of return and on taxation. Using the terminology of Kemp et al. the equilibrium can be characterized as partial feedback equilibrium because feedback interaction is taken into account only by the government (who considers the feedback effect of taxation on consumption). Note that – as in Shimomura (1991) – a

strategic interdependence occurs only through the state variable so that Markovian (i.e. feedback) Stackelberg and Nash solution coincide.<sup>7</sup>

Finally, note that capitalists are identical in the Judd–Kemp et al. modelling considered in this article. In particular, all capitalist share the same initial endowment. This assumption simplifies the analysis considerably because each capitalist can condition his consumption on the state of aggregate capital (of which he holds a constant share at all times).<sup>8</sup>

The steady-state  $(\tau^*, k^*, c_k^*)$  is determined by evaluating equations (1), (7), (13), and (14) at  $\dot{k} = \dot{c}_k = \dot{\tau} = 0$ . To solve the problem that at the steady-state both numerator and denominator of (14) are zero I apply l’Hôpital’s rule and get

$$(15) \quad \left( \frac{\partial c_k}{\partial k} \right)^* = \frac{\alpha(\alpha - 1)(1 - \tau)k^{\alpha-2}c_k}{\sigma [(1 - \tau)\alpha^2 k^{\alpha-1} - \delta - (\partial c_k / \partial k)^*]} .$$

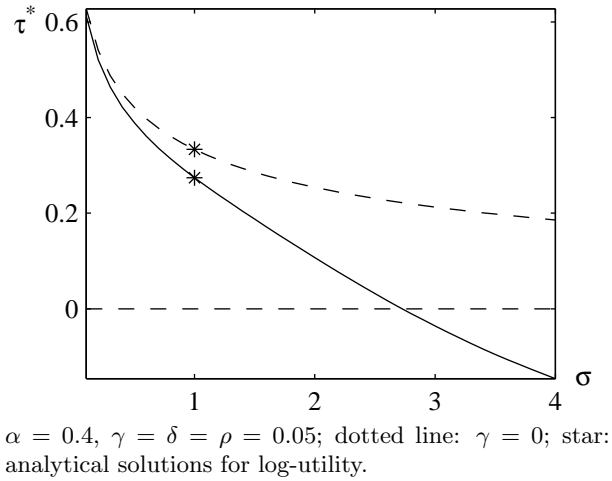
This quadratic equation has two solutions. Guessing from the standard Ramsey Model (Cass, 1965) that consumption is increasing in capital I take the positive one.

Figure 3 shows the correlation between optimal equilibrium tax rates and  $\sigma$  for an example economy with  $\alpha = 0.4$  and  $\delta = \rho = \gamma = 0.05$ . Dotted lines show results when the government acts solely in the interest of workers ( $\gamma = 0$ ). A star indicates the corresponding analytical solution for log-utility. Under Markovian strategies the somewhat strange result that the optimal tax rate abruptly changes (from zero to a positive value) as  $\sigma$  crosses one is no longer observable. The tax rate decreases continuously in  $\sigma$ . While  $\tau^*$  converges towards zero as  $\sigma$  goes to infinity for the case of  $\gamma = 0$ , it crosses the zero-line at about 2.7 if capitalists’ utility enters the social welfare function with a weight of 5 percent. Table 1 shows equilibrium tax rates for various combinations of  $\sigma$  and  $\gamma$ . For values of  $\sigma$  between 1 and 2 (values which are predominantly used in calibration of neoclassical growth models) and  $\gamma$  between zero and 5 percent, the example suggests equilibrium tax rates between 10 and 33 percent (values which are predominantly observed in reality).

<sup>7</sup>See Rubio (2003) for a general derivation and discussion of conditions for coincidence of feedback Stackelberg and Nash solution.

<sup>8</sup>See Lindner and Strulik (2004) for an investigation of Markovian tax strategies in Alesina and Rodrik’s (1994) linear growth model in which households are heterogenous with respect to initial wealth.

FIGURE 3. Optimal Tax Rates for Alternative  $\sigma$



A high value of  $\sigma$  indicates a strong preference to smooth consumption over time. An increase in capital taxes lowers net interest rates and induces a relatively strong intertemporal substitution effect. The incentive to tax is relatively small since the government knows that rising taxes would reduce investment considerably. If utility of capitalists enter the social welfare function with positive weight this may even imply a negative equilibrium tax rate as  $\sigma$  becomes large. On the other hand, the substitution effect is small for small values of  $\sigma$  and dominated by the income effect for  $\sigma < 1$ . Under these circumstances capitalists react to an initial decrease in income from capital taxation by investing more in subsequent periods and the incentive to tax is relatively strong for any government that can exploit this behavior as a Stackelberg leader.

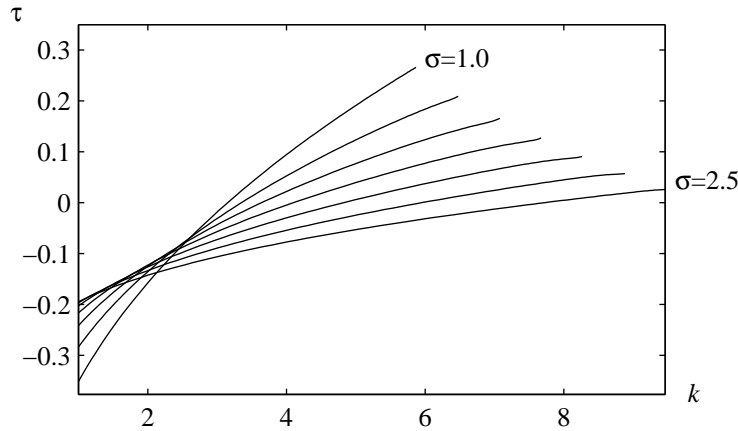
TABLE 1  
Optimal Equilibrium Capital Tax Rates

$\sigma$	$\gamma$	0.00	0.01	0.02	0.05	0.10
0.5		0.42	0.42	0.41	0.39	0.36
1.0		0.33	0.32	0.31	0.27	0.21
2.0		0.25	0.22	0.18	0.11	0.02
4.0		0.18	0.02	-0.05	-0.15	-0.23

$\alpha = 0.4, \delta = \rho = 0.05$

It remains to verify whether the result that government size increases in income per capita is robust when  $\sigma$  differs from one. For that purpose I integrate the system (1), (9), and (13) backwards starting close to the equilibrium  $(\tau^*, k^*, c^*)$  and use  $k = 1.0$  as termination criterion. Figure 4 shows the policy function  $\tau(k)$  for various values of  $\sigma$  between 1 and 2.5, i.e. it shows numerical equivalents of the stable manifold in Figure 1. For all values of  $\sigma$ , the tax is an increasing function of the capital stock and therewith positively correlated with income per capita. This positive relationship is less pronounced for high values of  $\sigma$  since the government takes into account the capitalists preference to smooth consumption over time. Figure 4 reveals a further interesting feature of the model. In contrast to the standard Ramsey model, the equilibrium capital stock depends on the intertemporal elasticity of substitution. This result reflects the fact that (as usual) the equilibrium capital stock depends on the capital tax rate and that (deviating from the standard approach) the capital tax rate is chosen optimally and time-consistently by the government and depends therefore on the tax payers' preference to smooth consumption over time.

FIGURE 4. Optimal Tax Policy  $\tau(k)$   
For Alternative Elasticities of Intertemporal Substitution



$\alpha = 0.4, \gamma = \delta = \rho = 0.05, \sigma \in \{1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2, 5\}$ .

## 5. CONCLUDING REMARKS

In this paper, I followed Meltzer and Richard's (1981) concluding appeal to integrate the struggle of income distribution into a model of capital accumulation and therewith to develop a "rational theory of the growth of government". For that purpose, I discussed adjustment dynamics in a simple model of capital taxation and redistribution where the government follows a time-consistent tax strategy. The simplicity of the model has permitted an analytical proof of the theory for an important special case. Generally, however, the occurrence of partial derivatives in the first order conditions of a Markovian-Stackelberg equilibrium prevents such an analytical investigation. One conceivable generalization has been investigated numerically by replacing log-utility with iso-elastic utility. It has been demonstrated that the theory of government growth is robust to this generalization.

In contrast to complementing studies, growth of the public sector has not been explained as a consequence of an inescapable "law" (Wagner, 1893) or a "disease" (Baumol, 1967), but as an optimal choice realized by a government acting in favor of its citizens by maximizing a social welfare function. The intuition for this result is also easy to convey. Given a general incentive to redistribute income from capital owners to workers and a government that cannot commit to its future policy, it is rational for the government to wait and refrain from capital taxation (or even subsidize capital income) at low stages of development when capital is scarce, capital productivity is high, and growth opportunities are large. At stages when capital is comparatively abundant and capital productivity is comparatively low it becomes desirable for the government to transfer income from capitalists to workers, enhance therewith social welfare, and manage a comparatively large public sector.

While the model can explain the positive correlation between relative government size and income per capita observable for all western countries for most of the last century, it fails to explain the decrease of government size observable for some countries (like e.g. the U.S.) during the last two decades. Increasing international tax competition constitutes one possible explanation for a secular decrease of capital tax rates. A verification of



this hypothesis in the framework presented could establish a two-country model where governments play Markovian Stackelberg strategies with their citizens and Markovian Nash strategies with each other. This represents an interesting extension for future research.

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